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Universal temperature dependence of the thermal conductivity and viscosity coefficients

A. S. OKHOTIN, L. I. ZHMAKIN and A. P. IVANYUK
A. N. Kosygin Moscow Textile Institute, Power Department, Moscow 117918, Russia, C.I.S.

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Abstract—Based on the relaxational model developed, the paper presents the mechanisms underlying the thermal conductivity and viscosity coefficients of substances in solid, liquid, and gaseous states. The analysis of the model made it possible to derive the temperature-varying thermal conductivity and viscosity coefficients. Their calculations from the proposed formulae are given for a number of well-known materials, and a good agreement with experimental results is shown.

THE MECHANISMS of heat transfer by heat conduction have been extensively studied by applying the models of a molecular real gas, electron gas, phonon gas, and a number of others. Different formulae derived on the basis of these models for the thermal conductivity coefficients can be found in a number of review papers [1–7]. All these models, and, consequently the formulae, are based on the interactions of particles of the gas considered with one another and with the defects of real substance in which heat transfer by conduction takes place. These mechanisms of interaction are conventionally called the processes of the scattering of gas particles (molecules, electrons or phonons), and they dominate in the description of the character of heat conduction and the numerical values of its coefficients. For the specialists dealing with the problems of the mechanism of heat conduction these are familiar problems that do not require further analysis within the scope of the present paper, the more so that the above-cited literature has given a considerable amount of attention to them. It should only be noted that all of the studies in this field do not allow one to obtain a single formula for the temperature dependence of the thermal conductivity coefficient of a certain class of substances from 0 K to the highest temperatures in the region of the gas state. It is known that the temperature dependence of the thermal conductivity coefficient for electrically conducting materials, referred to as the Wiedemann–Franz–Lorentz law, is most adequate, but it is also completely unsuitable in the region of low temperature ($T < \theta$ (the Debye temperature)) and in the region of the gas state. For solid bodies with $T > \theta$ and for a liquid phase, where this law is considered to be applicable, a considerable deviation from it is observed. The reasons for these deviations are also analysed in detail in the above-indicated literature and in other publications. As regards the formulae for the thermal conductivity coefficients of non-conducting solid bodies, liquids and gases, all of them are applicable either in a narrow temperature range and only for a certain small quantity of substances, or contain so many quantities and functions which are difficult to determine physically; all of these relations cannot claim a single mechanism of heat conduction and none can be accepted as a correlation.

In respect to the well-known expressions for the dynamic viscosity coefficients, it is to be noticed that these are still more empirical as they are based on the viscosity mechanisms that roughly approximate the real process. The formulae for determining the dynamic viscosity coefficients are presented in refs. [8–11]. None of the well-known physical quantities can be regarded as accurate enough for describing the temperature dependence of the viscosity coefficient in liquid and solid phases.

The foregoing has led the authors to attempt to devise a universal temperature dependence for the thermal conductivity and viscosity coefficients for a wide class of substances in the regions of solid, liquid and gaseous states. The attempts were based on studies conducted over many years in this area by one of the authors [2, 4, 12–24] and on the analysis of other numerous data published on this problem.

The present approach to the solution of the said problem is based on the well-known [25] classic relations for the thermal conductivity (λ) and dynamic viscosity (μ) coefficients for an ideal gas. These relations are presented in form more convenient for further analysis

$$\lambda = \tfrac{1}{3}c\rho v^2\tau_0 \tag{1}$$

and

$$\mu = \tfrac{1}{3}\rho v^2\tau_0 \tag{2}$$

where ρ is the density, c the heat capacity, v the velocity of gas particles and τ_0 the time for energy transfer adopted to be equal to 1 s in these equations. It is well known from molecular physics that it is impossible to use these simple relations for real substances owing to the complexity of calculations of the velocity and real time which may greatly differ from 1 s. The

NOMENCLATURE

c	heat capacity [J kg^{-1} K^{-1}]
h	Planck constant [J s]
$\hbar$	$h/2\pi$ [J s]
k	Boltzmann constant [J K^{-1}]
Pr	Prandtl number
T	temperature [K]
v	velocity [m s^{-1}].

Greek symbols

θ	Debye temperature [K]
λ	thermal conductivity [W m^{-1} K^{-1}]
μ	dynamic viscosity [Pa s]
ρ	density [kg m^{-3}]
τ	relaxation time [s].

Subscripts

boil	boiling
melt	melting
s	sound
*	characteristic state
$\bar{}$	averaging.

values of heat capacity and density have been well studied and are cited in numerous reference books. For a solid phase, especially at low temperatures, the values of the heat capacity are calculated from the Debye formula [26].

In order to determine the velocity and time dependence on temperature, the mechanisms of transfer in real bodies will be considered. First consideration will be given to the nature of the temperature dependence of the thermal conductivity (Fig. 1) and viscosity (Fig. 2) coefficients. Attention should be given to the fact that the magnitude of the thermal conductivity coefficient in the liquid and gas phases can both decrease and increase with an increasing temperature as depicted in Fig. 1 by two different lines for the liquid phase of the substance.

To describe this kind of temperature dependence of the thermal conductivity and viscosity coefficients, a universal model is proposed for heat transfer in a solid (metals, insulators, semiconductors), liquid and gaseous phase. The basis behind this model is that any body is considered to be a set of atoms that in the general case possess vibrational, rotational and translational motion and an electronic shell which is considered to be an independent particle in electrically conducting bodies. In the solid phase the atoms in monocrystalline materials are located at the lattice nodes with the corresponding long range order [27]. In polycrystalline and amorphous solid bodies the long range order is destroyed and the system preserves only the short range order with the models of disorder peculiar to it [28]. These models are also valid in the liquid phase. If the body in the solid and liquid phases has a metallic bond, then the energy of free electrons must be taken into account in the processes of heat transfer. It is considered that in the solid state the atoms have only vibrational and rotational motions; in the liquid phase the translational motion must be additionally taken into account when analysing the process of transfer. In the gaseous state the trans-

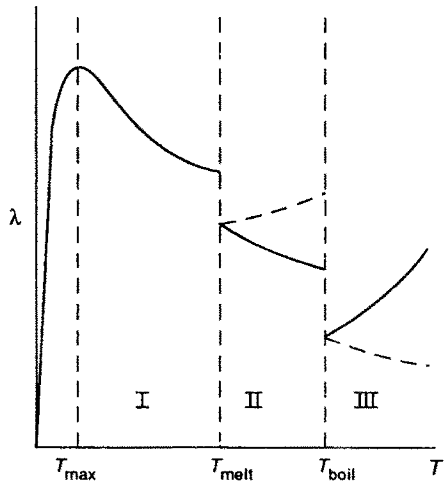

FIG. 1. The typical temperature dependence of the thermal conductivity coefficient: I, solid phase; II, liquid phase; III, gaseous phase.

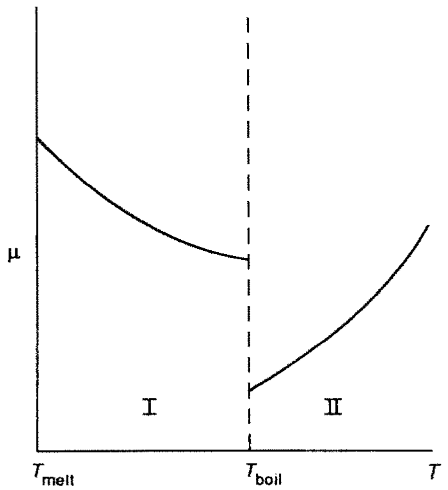

FIG. 2. The typical temperature dependence of the dynamic viscosity coefficient of substance: I, liquid phase; II, gaseous phase.

lational motion of atoms becomes predominant, the short range order disintegrates and the collectivization of electrons cannot be made. Consequently, in gases the atoms are considered to be the basic energy objects.

With such an approach to the composition of substance, the model suggested consists in the following: since the transfer of heat requires the application of the temperature gradient to the system, the heat supplied to the body is absorbed by the entire set of the particles, and because of the presence of the temperature gradient the energy absorbed propagates in the body in the form of a packet of waves. It is assumed that in any state of the substance the velocity of such a packet of waves is equal to the speed of sound. It is thus considered that the heat wave packet consists of a close set of waves having the frequencies of electrons, optical and acoustic phonons and that it displaces in the body with the speed of sound always as one wave, that is, it is presumed that no matter what particles are most energetic in the given substance, they do participate in the transfer of heat as a constituent part of the packet of heat waves. Then the specific features of heat transfer, for example, in metals or insulators, can be taken into account on the basis of the mutual scattering of particles inside the packet. Any substance in a solid, liquid or gaseous state can thus be represented as a set of waves (here the wave is understood to be the above-described packet of a heat wave), which, on application of the temperature gradient, move with the speed of sound in the direction opposite to the direction of the temperature gradient.

Since the speed of sound is inversely proportional to the compressibility of the medium, then the speed of sound in solid bodies and in incompressible liquids was assumed to be independent of temperature.

Moreover, the following mechanism of the scattering of waves is also assumed which determines the values of the thermal conductivity and viscosity. When moving in a solid body at very low temperatures ($T < T_{\max}$ in Fig. 1) a heat wave packet undergoes scattering only on the boundaries of the specimen, while at the temperatures $T_{\max} < T < \theta$ the scattering on the defects prevails. In both the cases it is necessary to take into account the in-packet scattering of all of the modes of vibrations in this heat wave packet, whereas at $T > \theta$ this form of scattering becomes predominant.

In amorphous bodies and liquids (melts including) the main form of scattering is considered to be the in-packet scattering of all the modes and the scattering of the heat wave packet on the disorder of coordinating spheres determined by the short range order in the given substance. The radii of coordinating spheres have been well studied in literature (see, e.g. ref. [29]). The basic form of scattering in a gaseous phase is considered to be the collisions of the heat wave packet with the atoms or with the associations of the atoms of the gas, with the in-packet scattering of all the vibrational modes being simultaneously taken into account.

All these forms of scattering are taken into account in the present model by the relaxation time quantity. It is based on the idea of the relaxation time in heat transfer by phonon heat conduction [30]. Developing this idea, the relaxation time of the heat wave packet for temperature T will be represented in the form

$$\tau = \frac{\hbar}{kT} \tag{3}$$

where $\hbar = h/2\pi$, h is the Planck constant and k the Boltzmann constant. The mean relaxation time for the given temperature T is suggested to be selected with allowance for the distribution of the packets of heat waves. This distribution can be represented as

$$n = \frac{1}{\exp(\tau/\tau^*) - 1} \tag{4}$$

where τ is the relaxation time in the form given by equation (3) and τ^* is the relaxation time at the characteristic temperature T^*

$$\tau^* = \frac{\hbar}{kT^*}. \tag{5}$$

The substitution of equations (3) and (5) into equation (4) yields

$$n = \frac{1}{\exp(T^*/T) - 1} \tag{6}$$

The minus sign in front of the unity in the denominator is due to the fact that, as already noted, any packet of heat waves in the model has a wide range of vibrational modes and consequently, obeys the Bose–Einstein statistics. With allowance for this fact, the mean relaxation time of the heat wave packet at temperature T has the form

$$\tau = \frac{\hbar/kT}{\exp(T^*/T) - 1}. \tag{7}$$

Of particular importance here is a correct determination of the characteristic temperature. When considering the heat conduction in solid bodies it is suggested to assume that $T^* = T_{\max}$, i.e. that this temperature corresponds to the maximum thermal conductivity coefficient, as shown in Fig. 1. For amorphous solid bodies $T^* = T_{\text{melt}}$. With this provision, the characteristic temperature is meant to denote the degree of the perfection of the crystal. For perfect monocrystals it approaches the zero temperature, and the higher the degree of imperfection, the higher $T_{\max}$ in amorphous bodies (complete disorder in the structure) $T_{\max}$ corresponds to the melting temperature.

When considering the process of heat conduction and internal friction (viscosity) in liquid and gaseous media, it is suggested that the characteristic temperature be the temperature of the phase change from a liquid to a gaseous state, i.e. in the majority of cases it is the boiling temperature, $T^* = T_{\mathrm{boil}}$. Then, for solid bodies equation (7) takes on the form

$$\bar{\tau} = \frac{\hbar/kT}{\exp(T_{\max}/T) - 1} \tag{8}$$

and for liquids and gases

$$\bar{\tau} = \frac{\hbar/kT}{\exp(T_{\mathrm{boil}}/T) - 1}. \tag{9}$$

Taking into account the above-described mechanisms of scattering of the packets of heat waves, it is suggested that the transition from an ideal gas to real bodies is made by introducing the correction for time τ_0 in equations (1) and (2) in the form

$$\bar{\tau}_{\mathrm{p}} = \left(\frac{\bar{\tau}}{\tau_0}\right)^{x}. \tag{10}$$

For the processes of heat transfer by conduction, with the representative scattering processes, it is taken that $x = n$ and then the correction will have the form

$$\bar{\tau} = \left(\frac{\hbar/kT}{\tau_0[\exp(T^*/T) - 1]}\right)^{n} \tag{11}$$

whereas for the processes of internal friction that determine the viscosity coefficients with its specific features of scattering it is taken that $x = m$, and the correction is

$$\bar{\tau}_{\mathrm{p}} = \left(\frac{\hbar/kT}{\tau_0[\exp(T_{\mathrm{boil}}/T) - 1]}\right)^{m}. \tag{12}$$

Summarizing the above (taking into account that $\tau_0 = 1$ s), it is thus possible, within the framework of the model, to suggest universal relations for solid, liquid and gaseous substances within the temperature range from 0 K to the highest temperatures that describe the temperature dependence of the thermal

Table 1. The values of n for alkali metals in a solid phase

T (K)	Li $T_{\max} = 16$ K	Na $T_{\max} = 4.5$ K	K $T_{\max} = 3$ K	Rb $T_{\max} = 4.5$ K	Cs $T_{\max} = 3.5$ K
1	0.297	0.346	0.377	0.436	0.465
3	0.454	0.457	0.494	0.553	0.585
5	0.513	0.514	0.570	0.613	0.652
7	0.547	0.562	0.630	0.659	0.697
9	0.571	0.605	0.675	0.696	0.726
10	0.582	0.624	0.693	0.711	0.737
12	0.600	0.656	0.722	0.734	0.751
15	0.624	0.696	0.755	0.759	0.766
20	0.658	0.745	0.791	0.780	0.780
30	0.715	0.805	0.822	0.797	0.792
40	0.761	0.834	0.832	0.804	0.798
50	0.797	0.849	0.837	0.807	0.801
100	0.877	0.866	0.844	0.812	0.808
300	0.910	0.869	0.848	0.815	0.813

Table 2. The values of n for alkali metals and mercury in liquid and gaseous phases

T (K)	Li $T_{\mathrm{boil}} = 1615$ K	Na $T_{\mathrm{boil}} = 1151$ K	K $T_{\mathrm{boil}} = 1032$ K	Rb $T_{\mathrm{boil}} = 959$ K	Cs $T_{\mathrm{boil}} = 943$ K	Hg $T_{\mathrm{boil}} = 630$ K
300	—	—	—	—	—	0.744
400	—	0.699	0.684	0.674	0.667	0.775
500	0.759	0.706	0.689	0.679	0.670	0.773
600	0.764	0.710	0.692	0.682	0.670	0.771
700	0.766	0.713	0.694	0.684	0.669	0.615
800	0.768	0.716	0.696	0.684	0.669	0.612
900	0.768	0.717	0.697	0.685	0.669	0.609
1000	0.768	0.719	0.699	0.650	0.644	0.606
1100	—	—	0.652	0.645	0.640	0.604
1200	—	0.651	0.648	0.641	0.636	—
1300	—	0.646	0.645	0.638	0.633	—
1400	—	0.641	0.642	0.635	—	—
1500	—	0.637	0.639	0.633	—	—
1600	—	—		—		—
1700	0.645	—	—	—	—	—
1800	0.645					
1900	0.644					
2000	0.643	—	—	—	—	—

Table 3. The values of n for electric conductive materials in a solid phase

T (K)	Cu $T_{\mathrm{max}} = 8.5$ K	Ag $T_{\mathrm{max}} = 7$ K	Au $T_{\mathrm{max}} = 9$ K	Be $T_{\mathrm{max}} = 35$ K	Mg $T_{\mathrm{max}} = 8.5$ K	Zn $T_{\mathrm{max}} = 6$ K	Cd $T_{\mathrm{max}} = 2.5$ K	Al $T_{\mathrm{max}} = 9$ K	Sn $T_{\mathrm{max}} = 2.3$ K	Pb $T_{\mathrm{max}} = 2.2$ K	Bi $T_{\mathrm{max}} = 4$ K
1	0.283	0.303	0.359	0.185	0.286	0.316	0.229	0.264	0.281	0.403	0.491
3	0.404	0.418	0.494	0.338	0.405	0.429	0.339	0.384	0.396	0.531	0.524
5	0.452	0.467	0.545	0.407	0.454	0.477	0.433	0.433	0.491	0.628	0.585
7	0.485	0.503	0.580	0.450	0.488	0.518	0.523	0.466	0.561	0.706	0.651
9	0.513	0.534	0.608	0.482	0.516	0.588	0.586	0.494	0.616	0.764	0.699
10	0.526	0.549	0.621	0.495	0.529	0.577	0.611	0.507	0.640	0.785	0.718
12	0.551	0.577	0.645	0.505	0.553	0.613	0.652	0.531	0.680	0.817	0.750
15	0.585	0.616	0.677	0.523	0.586	0.656	0.699	0.564	0.729	0.852	0.729
20	0.634	0.670	0.723	0.543	0.633	0.713	0.753	0.611	0.787	0.885	0.825
30	0.710	0.746	0.785	0.592	0.705	0.787	0.813	0.687	0.850	0.911	0.866
40	0.765	0.791	0.816	0.613	0.757	0.832	0.849	0.744	0.883	0.920	0.887
50	0.804	0.819	0.832	0.650	0.795	0.863	0.871	0.786	0.902	0.926	0.901
100	0.874	0.858	0.856	0.724	0.880	0.919	0.912	0.888	0.934	0.934	0.934
300	0.895	0.867	0.863	0.913	0.906	0.934	0.928	0.919	0.951	0.940	0.963
500	0.898	0.869	0.865	0.956	0.910	0.938	0.931	0.921	0.956	0.943	0.970
700	0.899	0.870	0.867	0.970	0.911	—	—	0.924	—	—	—
900	0.901	0.872	0.869	0.978	0.912	—	—	0.926	—	—	—
1100	0.902	0.874	0.871	0.985	—	—	—	—	—	—	—
1300	0.904	—	0.873	0.990	—	—	—	—	—	—	—

conductivity coefficient as

$$\lambda = \tfrac{1}{3}c\rho v_s^2\left(\frac{\hbar/kT}{\exp(T^*/T)-1}\right)^n \tag{13}$$

and of the dynamic viscosity coefficient as

$$\mu = \tfrac{1}{3}\rho v_s^2 = \left(\frac{\hbar/kT}{\exp(T_{\text{boil}}/T)-1}\right)^m. \tag{14}$$

Since the quantities c, ρ, v_s, h, k, T^*, and T_{boil} have been well studied for the majority of substances and have been tabulated or can be easily calculated from the data on the thermodynamic parameters, the main difficulty in using equations (13) and (14) consists in the correct determination of the values of n and m. Using the experimental data on the thermal conductivity and viscosity coefficients, the values of n and m were calculated from equations (13) and (14) for a great number of substances. In calculations for a solid phase, the values of ρ and v_s were assumed to be independent of temperature; the heat capacity was calculated from the Debye formula. The results of calculations are presented in Tables 1 to 11 from which it is seen that the values of n and m are the functions of temperature and in all of the cases they vary from 0 to 1. It seems that the essential point underlying these quantities is the allowance for the processes of scattering in real bodies in the course of energy transfer. By analogy with the effective scattering cross section that governs the transfer in ideal gases, the coefficients n and m can be regarded as being universal parameters of scattering, with the quantity n determining the scattering that governs the heat transfer and the quantity m determining the scattering that governs the origination of friction forces. The character of their temperature dependences is in good qualitative agreement with the above-described mechanism which underlies the scattering in solid, liquid and gaseous bodies which depends on the rise of temperature. The calculation of the quantities n and m for the present model seems to be possible with the use of the theory of fractiles [31, 32].

Table 4. The values of n for electrically conducting materials in a liquid phase

T (K)	Cu $T_{\text{boil}} = 2860$ K	Zn $T_{\text{boil}} = 1179$ K	Cd $T_{\text{boil}} = 1038$ K	Al $T_{\text{boil}} = 2593$ K	Sn $T_{\text{boil}} = 2540$ K	Pb $T_{\text{boil}} = 2023$ K
600	—	—	0.759	—	0.699	—
700	—	0.764	0.761	—	0.707	0.717
800	—	0.764	0.762	—	0.713	0.720
900	—	0.764	0.763	—	0.717	0.721
1000	—	0.764	0.763	0.744	0.720	0.721
1100	—	—	—	0.746	—	—
1200	—	—	—	0.748	—	—
1300	—	—	—	0.750	—	—
1400	0.727	—	—	0.751	—	—
1500	0.728	—	—	0.752	—	—
1600	0.729	—	—	—	—	—
1700	0.729	—	—	—	—	—
1800	0.730	—	—	—	—	—

Table 5. The values of n for semiconducting materials in a solid phase

T (K)	Si $T_{\max} = 24$ K	Ge $T_{\max} = 12$ K	Se (cryst.) $T_{\max} = 6.5$ K	Se (amorph.) $T_{\max} = 490.2$ K	Te $T_{\max} = 4.5$ K
2	0.366	0.499	0.507	—	0.445
5	0.446	0.526	0.571	—	0.531
8	0.477	0.573	0.627	—	0.602
10	0.492	0.598	0.661	0.345	0.640
12	0.506	0.618	0.691	0.392	0.671
15	0.522	0.646	0.726	0.452	0.709
20	0.550	0.684	0.768	0.531	0.754
30	0.597	0.740	0.818	0.636	0.804
40	0.640	0.779	0.844	0.700	0.832
50	0.682	0.807	0.862	0.741	0.850
100	0.780	0.876	0.898	0.825	0.894
300	0.888	0.938	0.933	0.849	0.934
500	0.945	0.982	—	—	—
1100	0.955	0.985	—	—	—
1300	0.960	—	—	—	—
1500	0.962	—	—	—	—

Table 6. The values of n for semiconducting materials in a liquid phase

T (K)	Si $T_{boil} = 3522$ K	Ge $T_{boil} = 3120$ K	Se $T_{boil} = 958$ K	Te $T_{boil} = 1663$ K
500	—	—	0.794	—
600	—	—	0.794	—
700	—	—	0.793	—
800	—	—	0.790	—
900	—	—	0.786	0.757
1000	—	—	—	0.760
1100	—	—	—	0.763
1200	—	—	—	0.766
1300	—	0.724	—	0.768
1400	—	0.726	—	—
1500	—	0.727	—	—
1700	0.750	—	—	—
1800	0.751	—	—	—
1900	0.753	—	—	—
2000	0.754	—	—	—

Table 7. The values of n for non-conducting materials in a gaseous phase

T (K)	N_2 $T_{boil} = 77.35$ K	O_2 $T_{boil} = 90.18$ K	H_2O $T_{boil} = 373.15$ K
65	0.938	—	—
70	0.939	—	—
75	0.939	0.943	—
80	—	0.943	—
85	—	0.944	—
90	—	0.944	—
273.15	—	—	0.906
283			0.907
293			0.908
303			0.909
313			0.910
323			0.911
333			0.912
343			0.913
353			0.914
363			0.915

Table 8. The values of n for non-conducting materials in a gaseous phase

T (K)	N_2 $T_{boil} = 77.35$ K	O_2 $T_{boil} = 90.18$ K	H_2O $T_{boil} = 373.15$ K
100	0.737	0.730	—
200	0.720	0.711	—
300	0.710	0.700	—
400	0.703	0.698	0.685
500	0.698	0.689	0.677
600	0.694	0.685	0.671
700	0.691	0.682	0.666
800	0.688	0.679	0.661
900	0.686	0.677	0.657
1000	0.684	0.674	0.654
1100	0.682	0.673	—
1200	0.679	0.670	—
1300	0.678	0.668	

Table 9. The values of m for alkali metals and mercury in liquid and gaseous phases

T (K)	Li $T_{boil} = 1615$ K	Na $T_{boil} = 1151$ K	K $T_{boil} = 1032$ K	Rb $T_{boil} = 959$ K	Cs $T_{boil} = 943$ K	Hg $T_{boil} = 630$ K
300	—	—	—	—	—	0.884
400	—	0.834	0.830	0.830	0.822	0.899
500	0.843	0.852	0.845	0.844	0.836	0.907
600	0.857	0.864	0.854	0.853	0.845	0.912
700	0.868	0.872	0.861	0.858	0.850	0.631
800	0.875	0.878	0.865	0.862	0.854	0.628
900	0.881	0.882	0.868	0.864	0.855	0.625
1000	0.886	0.855	0.870	0.655	0.655	0.622
1100	—	—	0.659	0.651	0.653	0.620
1200	—	0.644	0.657	0.649	0.651	—
1300	—	0.644	0.655	0.647	0.650	—
1400	—	0.644	0.654	0.645	—	—
1500	—	0.643	0.652	0.643	—	—
1600	—	—	—	—	—	—
1700	0.652					
1800	0.652					
1900	0.652					
2000	0.652	—	—	—	—	—

Table 10. The values of *m* for non-conducting materials in a liquid phase

T (K)	N_2 $T_{boil} = 77.35$ K	O_2 $T_{boil} = 90.18$ K	H_2O $T_{boil} = 373.15$ K
65	0.906	—	—
70	0.910	—	—
75	0.915	0.909	—
80	—	0.913	—
85	—	0.917	—
90	—	0.920	—
273.15	—	—	0.827
283			0.838
293			0.848
303			0.856
313			0.864
323			0.871
333			0.878
343			0.884
353			0.889
363			0.894

Equations (13) and (14) can easily yield the expression for the Prandtl number which, on the basis of the present model, will eventually have the form

$$Pr = \tau^{m-n}. \tag{15}$$

Calculations of the Prandtl number from this equation for alkali melts, mercury, water, nitrogen, and oxygen satisfactorily coincide with the well-known literature data.

Table 11. The values of *m* for non-conducting materials in a gaseous phase

T (K)	N_2 $T_{boil} = 77.35$ K	O_2 $T_{boil} = 90.18$ K	H_2O $T_{boil} = 373.15$ K
100	0.746	0.738	—
200	0.730	0.721	—
300	0.721	0.712	—
400	0.715	0.705	0.685
500	0.710	0.700	0.678
600	0.706	0.696	0.673
700	0.703	0.692	0.669
800	0.700	0.689	0.665
900	0.697	0.687	0.661
1000	0.695	0.685	0.658
1100	0.693	0.683	—
1200	0.691	0.681	—
1300	0.689	0.679	

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DEPENDANCE UNIVERSELLE DES COEFFICIENTS DE CONDUCTIVITE THERMIQUE ET DE VISCOSITE VIS-A-VIS DE LA TEMPERATURE

Résumé—A partir d'un modèle relaxationnel, on présente les mécanismes sous-tendant les coefficients de conductivité thermique et de viscosité des substances dans les états solide, liquide et gazeux. L'analyse du modèle rend possible l'obtention de la loi de dépendance de ces coefficients vis-à-vis de la température, leur calcul à partir des formules proposées sont données pour de nombreux matériaux connus et on constate un bon accord avec les résultats expérimentaux.

UNIVERSELLE TEMPERATURABHÄNGIGKEIT DER WÄRMELEITFÄHIGKEIT UND DER VISKOSITÄT

Zusammenfassung—Auf der Grundlage eines bereits entwickelten Relaxationsmodells wird in der vorliegenden Arbeit über den Mechanismus berichtet, auf dem die Wärmeleitfähigkeit und die Viskosität fester, flüssiger und gasförmiger Substanzen beruht. Die Analyse des Modells machte es möglich, die temperaturabhängige Wärmeleitfähigkeit und Viskosität herzuleiten. Berechnungen dieser beiden Koeffizienten werden für einige bekannte Stoffe angegeben, wobei sich eine gute Übereinstimmung mit Versuchsdaten zeigt.

УНИВЕРСАЛЬНАЯ ЗАВИСИМОСТЬ КОЭФФИЦИЕНТОВ ТЕПЛОПРОВОДНОСТИ И ВЯЗКОСТИ ОТ ТЕМПЕРАТУРЫ

Аннотация—В статье излагаются механизмы коэффициентов теплопроводности и вязкости в твердом, жидком и газообразном состояниях на основе разработанной релаксационной модели. Приводятся полученные из анализа этой модели температурные зависимости коэффициентов теплопроводности и вязкости. Приведены их расчеты по предложенным формулам для ряда хорошо изученных материалов и показано хорошее совпадение с экспериментальными результатами.